STAT 350 Exam 1 SPRING 2026

V1

Name: _____	PUID _____

Instructor (circle one): **Heekyung Ahn** **Yu Lin** **Evidence Matangi** **Timothy Reese** **Halin Shin** **Chenzhong Wu**

Select Class Meeting Days/Time

- ○ **MWF** 10:30AM-11:20PM
- ○ **MWF** 11:30AM-12:20PM
- ○ **MWF** 12:30 PM-1:20PM
- ○ **MWF** 1:30-2:20PM
- ○ **MWF** 2:30PM-3:20PM
- ○ **MWF** 3:30 PM-4:20PM
- ○ **T/TH** 1:30 PM-2:45PM
- ○ **T/TH** 3:00PM-4:15PM
- ○ **Online**

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

Instructions:

1. Please write your **name** and **PUID clearly** on every **odd page.**
2. **Write your work in the box. Do not run over into the next question space.**
3. The only materials that you are allowed during the exam are your **scientific calculator**, **writing utensils, erasers, your crib sheet**, and **your picture ID**. Colored scratch paper will be provided if you need more room for your answers. Please write your name at the top of that paper also.
4. The crib sheet can be a handwritten or typed double-sided 8.5in x 11in sheet.
5. If you share your calculator without permission or have a cell phone at your desk, you will get a **zero** on the exam. Do not take out your cell phone until you are next in line to submit your exam.
6. The exam is only 60 minutes long so there will be no breaks during the exam. If you leave the exam room, you must turn in your exam, and you will not be allowed to come back.
7. **For free response questions you must show ALL your work to obtain full credit.** An answer without showing any work may result in **zero** credit. If your work is not readable, it will be marked wrong. Remember that work must be shown for all numbers that are not provided in the problem or no credit will be given for them. All explanations must be in complete English sentences to receive full credit.
8. All numeric answers should have **four decimal places** unless stated otherwise.
9. After you complete the exam, please turn in your exam as well as your table and any scrap paper that you used. Please be prepared to **show your Purdue picture ID**.
10. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone caught cheating on the exam will automatically fail the course and will be reported to the Office of the Dean of Students.
11. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you on Gradescope after it is graded.

Your exam is not valid without your signature below. This means that it won't be graded.

I attest here that I have read and followed the instructions above honestly while taking this exam and that the work submitted is my own, produced without assistance from books, other people (including other students in this class), notes other than my own crib sheet(s), or other aids. In addition, I agree that if I tell any other student in this class anything about the exam BEFORE they take it, I (and the student that I communicate the information to) will fail the course and be reported to the Office of the Dean of Students for Academic Dishonesty.

Signature of Student: _____

You may use this page as scratch paper.
The following is for your benefit only.

Question Number	Total Possible	Your points
Problem 1 (True/False) (2 points each)	12	
Problem 2 (Multiple Choice) (3 points each)	15	
Problem 3	15	
Problem 4	26	
Problem 5	17	
Problem 6	20	
Total	105	

The rest of this page can be used for scratch work

1. **(12 points, 2 points each) True/False Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect.

1.1. Let $\boldsymbol{X}$ denote a continuous random variable with a PDF $\boldsymbol{f_X(x)}$. For any interval such that $[\boldsymbol{a, b}] \subset \mathbf{Support(X)}$, such that $\boldsymbol{a < b}$,

Ⓣ or Ⓕ $\boldsymbol{P(a < X < b)}$ must be less than or equal to $\boldsymbol{P(X < b)}$**.**

1.2. Suppose $\boldsymbol{X}$ is a Binomial random variable with parameters $\boldsymbol{n}$ and $\boldsymbol{p}$.

Ⓣ or Ⓕ Holding the number of trials $\boldsymbol{n}$ constant, the shape of the distribution shifts from positively skewed to negatively skewed as $\boldsymbol{p}$ changes from **0.9** to **0.1**.

1.3. Regarding the properties of a Binomial random variable $\boldsymbol{X \sim \mathbf{Binomial(n, p)}}$.

Ⓣ or Ⓕ The variance of $\boldsymbol{X}$ cannot exceed the number of independent trials $\boldsymbol{n}$.

1.4. Suppose $\boldsymbol{V \sim \mathbf{Binomial(n, p)}}$ and $\boldsymbol{W \sim \mathbf{Poisson}(\lambda)}$.

Ⓣ or Ⓕ Then for any positive integer $\boldsymbol{n}$, the support of $\boldsymbol{V}$ is a subset of the support of $\boldsymbol{W}$**.**

1.5. When converting a value $\boldsymbol{x}$ from a normal distribution into a $\boldsymbol{z}$**-score**.

Ⓣ or Ⓕ A negative $\boldsymbol{z}$**-score** indicates that the original x is smaller than the population mean $\boldsymbol{\mu}$.

1.6. Suppose $\boldsymbol{X}$ and $\boldsymbol{Y}$ are Normally distributed random variables sharing the same mean $\boldsymbol{\mu = 10}$. It is also known that $\mathbf{Var(X) < Var(Y)}$.

Ⓣ or Ⓕ Then it follows that $\boldsymbol{P(X \leq 12)}$ is larger than $\boldsymbol{P(Y \leq 12)}$.

2. **(15 points, 3 pts each) Multiple Choice Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect. **For each question, there is only one correct option letter choice unless specified.**

2.1. Let $\boldsymbol{X}$ and $\boldsymbol{Y}$ be discrete random variables that are **not** independent. Choose the statement about $\boldsymbol{X}$ and $\boldsymbol{Y}$ that always holds.

Ⓐ $\mathbf{Var}(\boldsymbol{X + Y}) = \mathbf{Var}(\boldsymbol{X}) + \mathbf{Var}(\boldsymbol{Y})$

Ⓑ For any $\boldsymbol{x} \in \mathbf{Support}(\boldsymbol{X})$ and $\boldsymbol{y} \in \mathbf{Support}(\boldsymbol{Y})$, $\boldsymbol{P(X = x, Y = y) = P(X = x)P(Y = y)}$.

Ⓒ $\boldsymbol{E[XY] = E[X]E[Y]}$

Ⓓ $\boldsymbol{E[X^3 + Y^{-2}] = E[X^3] + E[Y^{-2}]}$

Ⓔ $\mathbf{Cov}(\boldsymbol{X, Y}) > \mathbf{0}$

2.2. On days when STAT 350 homework is due, suppose Professor Reese receives extension requests according to a Poisson process at an average rate of 0.05 requests per 10 minutes. Compute the probability that he receives more than 1 extension request in a randomly selected 2-hour period.

Ⓐ 0.0012

Ⓑ 0.0488

Ⓒ 0.1219

Ⓓ 0.4512

Ⓔ 0.8781

2.3. For some constant $\boldsymbol{k}$, define a PDF $\boldsymbol{f_X(x) = k \cdot (x-5)^2}$ for $\boldsymbol{x \in [4, 6]}$ and zero elsewhere. Which of the following statements correctly describes this distribution?

Ⓐ The distribution is bimodal.

Ⓑ The distribution is positively skewed.

Ⓒ The normalizing constant $\boldsymbol{k}$ can be negative.

Ⓓ The median is larger than the mean.

Ⓔ None of the above statements correctly describes the distribution.

2.4. The weights of packages shipped from a warehouse are Normally distributed with mean $\boldsymbol{\mu = 50}$ pounds (lbs) and standard deviation $\boldsymbol{\sigma = 4}$ lbs. A package is considered "light" if its weight is in the bottom 2.5% of the distribution. What is the cutoff weight to be considered a "light" package?

Ⓐ 38 lbs

Ⓑ 40 lbs

Ⓒ 42 lbs

Ⓓ 44 lbs

Ⓔ 46 lbs

Ⓕ 48 lbs

2.5. Let $\boldsymbol{T}$ represent the "Triage Window" (in minutes) for resolving IT tickets at Purdue university. Suppose $\boldsymbol{T}$ follows a Normal distribution where it is known that $\boldsymbol{P(T > 15) = 0.0668}$ and $\boldsymbol{P(T < 5) = 0.1587}$. What is the mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$ of this distribution?

Ⓐ $\boldsymbol{\mu = 8, \sigma = 3.5}$

Ⓑ $\boldsymbol{\mu = 9, \sigma = 4}$

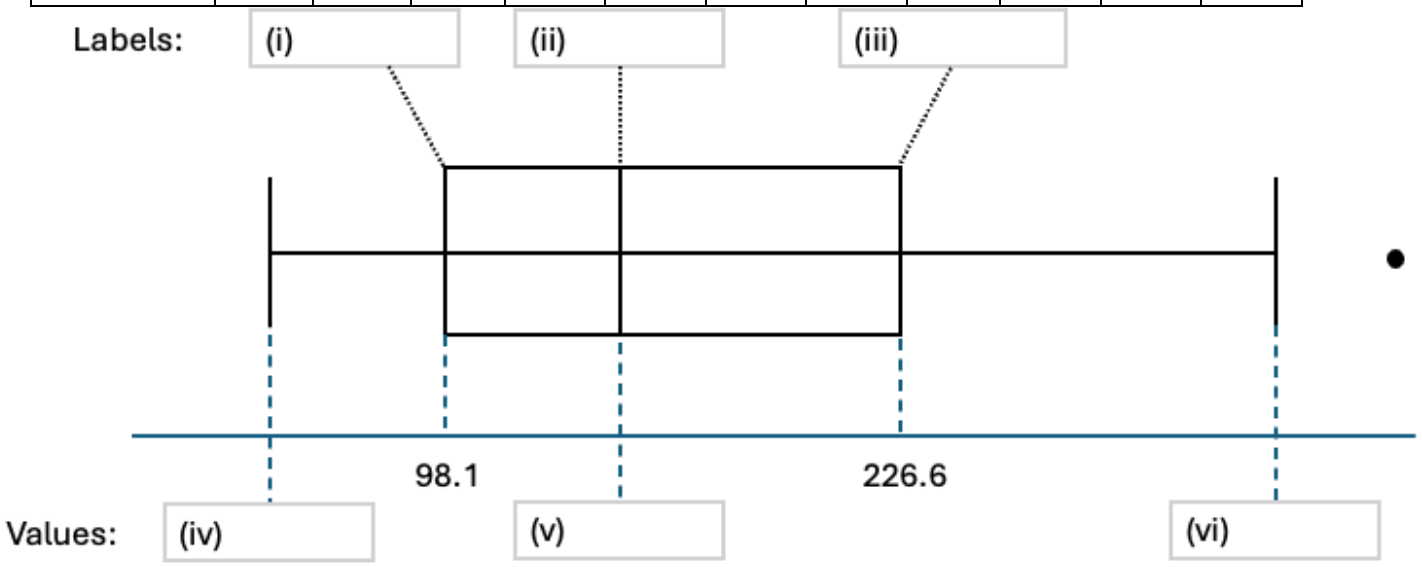
Ⓒ $\boldsymbol{\mu = 10, \sigma = 2}$

Ⓓ $\boldsymbol{\mu = 10, \sigma = 5}$

Free Response Questions 3-5. Show all work, clearly label your answers, and use **four decimal places**.

3. **(15 points)** A psychological research group studies the change in university students' screen time and how this affects their studying patterns. As part of the study, they collected the screen time, in minutes, of 47 students. Below is a partial data table containing the sorted observations and a corresponding partial modified boxplot.

Index	1	2	⋯	23	24	25	26	⋯	45	46	47
Observation	42.5	54.4	⋯	132.0	145.3	147.9	158.2	⋯	335.6	342.8	487.2



a) **(6 points)** Fill in the blank spaces **(i) – (vi)** corresponding to the boxplot. For boxes **(i), (ii),** and **(iii)**, provide the correct statistical terminology. For boxes **(iv), (v),** and **(vi),** provide the exact numerical value.

b) (3 points) Based on the distribution shown, where is the mean for this dataset most likely to exist?

Ⓐ Between the minimum and first quartile.

Ⓑ Between the first quartile and the median.

Ⓒ Between the median and third quartile.

Ⓓ Between the third quartile and maximum.

Ⓔ No single option is more likely than others.

c) (3 points) Compute the interquartile range (IQR). Explain its significance strictly in the context of the students' screen time data.

d) (3 points) Approximately how many data points are at least 98.1 and at most 226.6?

4. **(26 points)** Due to recent severe mechanical failures on the assembly line, a production facility is experiencing an unusually high rate of errors. A quality inspector examines a small batch of $\boldsymbol{n = 4}$ electronic components from a massive production line. Each component independently has a probability $\boldsymbol{p = 0.3}$ of being defective. Let $\boldsymbol{X}$ denote the number of defective components in the batch.
   a) **(4 points)** Identify the distribution of $\boldsymbol{X}$, including its **parameter(s)**. Write out the exact **PMF formula f**or $\boldsymbol{P(X = x)}$ and state the **support** of $\boldsymbol{X}$.

   b) **(5 points)** Compute the probability of observing **at least two defective components** in the batch.

c) (5 points) Determine the expected number of **defective components**, the expected number of **non-defective components,** and the variance.

d) (12 points) Suppose the automated QA machine scans the batches. If a batch has many defects, the automated QA machine halts early and rejects it. The diagnostic time (in minutes) spent on a batch is modeled by the function $\boldsymbol{D} = \frac{\boldsymbol{60}}{\boldsymbol{X+1}}$. Calculate the expected diagnostic time, $\boldsymbol{E}[\boldsymbol{D}]$. (Hint: The LOTUS flower brings clarity.)

5. **(17 points)** Meredith 😺 follows a daily routine in the following order: eat → drink → poop → cuddle → sleep. If Meredith successfully completes the first four steps, she falls asleep and is happy 😺. If any of the steps are broken, Meredith is guaranteed to get mad 😾.

   Let $(\boldsymbol{M})$ be the event that Meredith gets mad 😾; otherwise she is happy $(\boldsymbol{H})$ 😺 (falls asleep). Let $\boldsymbol{E}$, $\boldsymbol{D}$, $\boldsymbol{P}$, and $\boldsymbol{C}$ be the events that the routine is broken at the **eat, drink, poop,** and **cuddle** step, respectively. From an observational study, Meredith's owner, Heekyung, learned that $\boldsymbol{P(M) = 0.2}$. When Meredith gets mad, the cause of the broken routine is $\boldsymbol{50\%}$ **eat**, $\boldsymbol{25\%}$ **drink**, $\boldsymbol{10\%}$ **poop**, and $\boldsymbol{15\%}$ **cuddle**.

   a) **(5 points)** What is the probability that Meredith gets mad and the broken routine is "poop"?

   b) **(5 points)** What is the probability that the broken routine is "eat" given that Meredith is happy?

   c) **(5 points)** What is the probability that Meredith gets mad given that the "poop" routine is broken?

d) (2 points) Determine whether the events $\boldsymbol{M}$ and $\boldsymbol{P}$ are independent or not.

Ⓐ The two events are independent.

Ⓑ Two events are dependent.

6. (20 points) A data science lab is running a mandatory 10-hour thermal stress test on a new cluster of machine learning GPUs. Let $\boldsymbol{T}$ be the time (in hours) until a defective GPU fails during the test.

- Phase 1 $(\mathbf{0} \leq \boldsymbol{t} \leq \mathbf{1})$: The thermal load ramps up linearly for the first hour.
- Phase 2 $(\mathbf{1} < \boldsymbol{t} \leq \mathbf{10})$: The probability of failure decays smoothly according to an inverse-square law.
- The Cutoff: The stress test is automatically halted at exactly 10 hours.

The probability density function (PDF) for the failure time is modeled by a continuous piecewise function:

$$f_X(x) = \begin{cases} \frac{5}{7} \cdot t & 0 \leq t \leq 1 \\ \frac{5}{7} \cdot \frac{1}{t^2} & 1 < t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

a) (10 points) The partially completed cumulative distribution function (CDF) is given below. Find the missing equation for the region between 1 and 10.

$$F_X(x) = \begin{cases} 0 & t < 0 \\ \frac{5}{14} \cdot t^2 & 0 \leq t < 1 \\ [\text{MISSING}] & 1 \leq t < 10 \\ 1 & t \geq 10 \end{cases}$$

b) (10 points) Calculate the median failure time for a defective GPU.

You may use this page as scratch paper

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998