

STAT 350 Exam 1 FALL 2026

V1

Name: _____ PUID _____

Instructor (circle one):

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Evidence Matangi **Timothy Reese**

Select Class Meeting Days/Time

○ **MWF** 9:30AM-10:20AM ○ **MWF** 1:30PM-2:20PM
○ **MWF** 10:30AM-11:20AM ○ **MWF** 2:30 PM-3:20PM
○ **MWF** 11:30 PM-12:20PM ○ **T/TH** 12:00 PM-1:15PM
○ **MWF** 12:30PM-1:20PM ○ **T/TH** 1:30PM-2:45PM ○ **Online**

Exam Instructions

1. **Identification:** Write your last name clearly on every odd page and on any provided scratch paper. Have your Purdue picture ID ready when submitting the exam.
2. **Allowed Materials:** Scientific calculator, writing utensils, erasers, and one double-sided 8.5" x 11" crib sheet (handwritten or typed).
3. **Formatting & Work:** Keep all work strictly within the designated boxes. Round all numeric answers to **four decimal places** unless stated otherwise.
4. **Grading Requirements:** You must show ALL work for free-response questions. Unsupported numbers, unreadable work, or missing explanations (which must be in complete English sentences) may receive zero credit.
5. **Logistics:** The exam is 60 minutes. Leaving the room finalizes your submission. After you complete the exam, please turn in your exam as well as any scrap paper that you used. Please be prepared to **show your Purdue picture ID**. Exams will be returned via Gradescope.

Strict Prohibitions & Honor Code

- **Zero Tolerance:** Absolutely no cell phones at your desk. No sharing calculators. Violating either rule may result in an immediate zero on the exam.
- **Academic Integrity:** You are responsible for keeping your work covered at all times. The unauthorized removal of exam materials from the room, any form of academic dishonesty, or discussing exam contents with students who have not yet tested may result in a zero on the exam and potentially a formal report to the Office of the Dean of Students.

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

Your exam is not valid without your signature below. This means that it won't be graded.
I attest that I have read and followed the instructions above honestly. The work submitted is my own, produced without unauthorized assistance. I agree that if I share information about this exam with any student before they take it, both parties will fail the course and be reported for Academic Dishonesty.

Signature of Student: _____

You may use this page as scratch paper.
The following is for your benefit only.

Question Number	Total Possible	Your points
Problem 1 (True/False) (2 points each)	12	
Problem 2 (Multiple Choice) (3 points each)	15	
Problem 3	26	
Problem 4	26	
Problem 5	26	
Total	105	

The rest of this page can be used for scratch work

1. **(12 points, 2 points each) True/False Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect.

1.1. A dataset $\boldsymbol{X}$ has an interquartile range (IQR) of $\mathbf{IQR}_{\boldsymbol{X}}$. Each value in the dataset is multiplied by −2, and then 10 is added, giving new values $y = -2x + 10$.

Ⓣ or Ⓕ The IQR of the new values of the dataset $\boldsymbol{Y}$ is $-2 \cdot \mathbf{IQR}_{\boldsymbol{X}} + 10$.

1.2. Let A and B be two non-empty events from the same sample space Ω, both with positive probabilities ($\boldsymbol{P(A) > 0}$ and $\boldsymbol{P(B) > 0}$).

Ⓣ or Ⓕ If $\boldsymbol{A}$ and $\boldsymbol{B}$ are mutually exclusive (disjoint), then they cannot be independent events.

1.3. Let $\boldsymbol{X}$ be a discrete random variable with finite mean μ and standard deviation $\boldsymbol{\sigma > 0}$. Define a new random variable $\boldsymbol{Y = 4 - 3X}$.

Ⓣ or Ⓕ The standard deviation of $\boldsymbol{Y}$ is equal to $\boldsymbol{-3\sigma}$.

1.4. Let $\boldsymbol{X}$ be a Binomial random variable with $\boldsymbol{n \geq 1}$ independent Bernoulli trials and success probability $\boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$.

Ⓣ or Ⓕ The variance $\mathbf{Var}(\boldsymbol{X})$ can never exceed $\frac{\boldsymbol{n}}{\mathbf{4}}$.

1.5. Let X be a continuous random variable with probability density function $\boldsymbol{f_X(x)}$.

Ⓣ or Ⓕ The density must satisfy $\boldsymbol{f_X(x) \leq 1}$ for all $\boldsymbol{x} \in \mathbb{R}$.

1.6. Suppose $\boldsymbol{X \sim N(\mu, \sigma^2)}$.

Ⓣ or Ⓕ The probability that $\boldsymbol{X}$ falls within 1.28 standard deviations of the mean, $\boldsymbol{P(\mu - 1.28\sigma \leq X \leq \mu + 1.28\sigma)}$, is 0.80.

2. **(15 points, 3 pts each) Multiple Choice Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect. **For each question, there is only one correct option letter choice unless specified.**

2.1. A survey of 150 recent Purdue engineering graduates revealed that their starting annual salaries had a sample mean of $\$86{,}500$ and a sample median of $\$78{,}000$. Based on these summary statistics, which of the following statements is most likely true regarding the distribution of starting salaries?

Ⓐ The distribution is symmetric and mound-shaped because engineering salaries are standardized.

Ⓑ The distribution is right-skewed (positively skewed) because a relatively small number of exceptionally high salaries pull the sample mean above the sample median.

Ⓒ The distribution is left-skewed (negatively skewed) because the majority of graduates earn less than the mean.

Ⓓ The sample standard deviation must be equal to zero.

Ⓔ The interquartile range (IQR) is guaranteed to be larger than the full range of the data.

2.2. During peak lunch hours at the Purdue Memorial Union, mobile food orders arrive according to a Poisson process at an average rate of 0.4 orders per minute. What is the probability that exactly 2 orders arrive in a randomly selected 5-minute interval?

Ⓐ 0.0544

Ⓑ 0.1465

Ⓒ 0.2707

Ⓓ 0.4000

Ⓔ 0.6767

2.3. Let $\boldsymbol{X}$ be a discrete random variable representing the number of defective components found in an electronic assembly pack, with probability mass function:

$$\boldsymbol{P(X=0)=0.60,\quad P(X=1)=0.30,\quad P(X=2)=0.10}$$

The repair cost (in dollars) for a pack is modeled by the function $\boldsymbol{C(X)=50+20X^2}$. What is the expected repair cost $\boldsymbol{E[C(X)]}$?

Ⓐ $\$50.00$

Ⓑ $\$57.00$

Ⓒ $\$60.00$

Ⓓ $\$64.00$

Ⓔ $\$78.00$

2.4. An inspector examines a small shipment of $\boldsymbol{N=25}$ laptops. Unbeknownst to the inspector, the shipment contains **exactly** $\boldsymbol{4}$ defective units. The inspector randomly selects $\boldsymbol{n=5}$ laptops without replacement and counts the number of defective units $\boldsymbol{X}$ among the $\boldsymbol{5}$ **sampled**. Which of the following explains why $\boldsymbol{X}$ does NOT strictly follow a Binomial distribution?

Ⓐ The draws are dependent because each selection changes which laptops remain in the shipment.

Ⓑ A draw cannot have binary outcomes because defect status is learned only after inspection.

Ⓒ The number of trials is not fixed because the defect count is unknown before inspection.

Ⓓ The expected defect count cannot be computed before the inspector checks the selected laptops.

Ⓔ The variance must exceed the sample size because the inspector samples without replacement.

2.5. The delivery time $\boldsymbol{T}$ (in minutes) for autonomous delivery robots on the Purdue campus follows an Exponential distribution with rate $\boldsymbol{\lambda = 0.10}$ per minute. The dining service promises a delivery fee refund if an order takes longer than $\boldsymbol{c}$ minutes. If they wish to issue refunds for only the top 2.5% slowest deliveries, what should the threshold $\boldsymbol{c}$ be?

Ⓐ 27.73 minutes

Ⓑ 29.96 minutes

Ⓒ 36.89 minutes

Ⓓ 39.12 minutes

Ⓔ 46.05 minutes

Free Response Questions 3-5. Show all work, clearly label your answers, and use **four decimal places**.

3. **(26 points)** A High-Performance Computing (HPC) cluster at Purdue processes large-scale scientific simulations. Every submitted task runs a mandatory one-hour CPU calibration stage before its simulation can finish or reach a checkpoint. Let $\boldsymbol{X}$ denote the total active CPU runtime (in hours), including that calibration stage, of a randomly submitted simulation task before completion or checkpointing. Thus, $\boldsymbol{X}$ cannot be less than one hour. The probability density function (PDF) of $\boldsymbol{X}$ is modeled by the following piecewise function:

$$\boldsymbol{f_X(x)} = \begin{cases} \boldsymbol{cx}, & \boldsymbol{1 \le x \le 3} \\ \boldsymbol{3c}, & \boldsymbol{3 < x \le 5} \\ \boldsymbol{0}, & \textbf{otherwise} \end{cases}$$

   a) **(8 points)** Determine the value of the constant c such that $\boldsymbol{f_X(x)}$ is a valid probability density function. Show all integration steps clearly.

b) (6 points) The intervals of the Cumulative Distribution Function $F_X(x)$ are shown below. Find the expression for each labeled blank (i)–(iv) to complete $F_X(x)$ for all real numbers $x \in \mathbb{R}$.

$$F_X(x) = \begin{cases} \boxed{\text{(i)}} & x < 1 \\ \boxed{\text{(ii)}} & 1 \le x \le 3 \\ \boxed{\text{(iii)}} & 3 < x \le 5 \\ \boxed{\text{(iv)}} & x > 5 \end{cases}$$

c) (4 points) What is the probability that a randomly chosen simulation task takes between 2 and 4 hours to finish?

d) (4 points) Given that a simulation task has already run for at least 2 hours, what is the conditional probability that it takes more than 4 hours to finish?

e) (4 points) Find the median execution duration $\tilde{\mu}$ (in hours) of a simulation task.

4. (26 points) Purdue's autonomous electric shuttle fleet navigates three campus routes: Route $\boldsymbol{A}$ (Engineering Quad), Route $\boldsymbol{B}$ (Stadium Drive), and Route $\boldsymbol{C}$ (Discovery Park).

Shuttles are assigned randomly to routes based on campus demand:

- Route $\boldsymbol{A}$: assigned 50% of the time, $\boldsymbol{P(A) = 0.50}$

- Route $\boldsymbol{B}$: assigned 30% of the time, $\boldsymbol{P(B) = 0.30}$

- Route $\boldsymbol{C}$: assigned 20% of the time, $\boldsymbol{P(C) = 0.20}$

During operation, high pedestrian density or sun glare may trigger a safety sensor alert ($\boldsymbol{S}$). Telemetry data shows:

- On Route $\boldsymbol{A}$: $\boldsymbol{P(S \mid A) = 0.08}$

- On Route $\boldsymbol{B}$: $\boldsymbol{P(S \mid B) = 0.15}$

- On Route $\boldsymbol{C}$: $\boldsymbol{P(S \mid C) = 0.25}$

a) (4 points) Calculate the joint probabilities for the three alert scenarios: $\boldsymbol{P(A \cap S)}$, $\boldsymbol{P(B \cap S)}$, **and** $\boldsymbol{P(C \cap S)}$.

b) (6 points) Calculate the overall probability $\boldsymbol{P(S)}$ that a randomly selected shuttle trip experiences a sensor alert.

c) (8 points) Suppose a shuttle trip is known to have experienced a sensor alert ($\boldsymbol{S}$). Compute the probability that this shuttle was traveling on Route $\boldsymbol{A}$.

d) (5 points) Are the event {Shuttle is on Route A} and the event {Sensor alert occurs} independent? State YES or NO and provide clear mathematical justification.

e) (3 points) Suppose 4 independent shuttle trips on Route C are monitored. What is the probability that at least one of these 4 trips experience a sensor alert?

5. (26 points) High-precision silicon wafer substrates manufactured at the Birck Nanotechnology Center have a thickness X (in micrometers) that follows a Normal distribution with unknown mean μ and unknown standard deviation σ.

Quality assurance testing on a large production run establishes the following two specifications:

- Exactly 10% of the manufactured wafers have a thickness strictly less than 47.812 micrometers:

$$P(X < 47.812) = 0.10$$

- Exactly 5% of the manufactured wafers have a thickness strictly greater than 53.944 micrometers:

$$P(X > 53.944) = 0.05$$

Use a standard normal table with z values rounded to two decimal places. **If two table entries are equally close to a target cumulative area, use the smaller z value.**

a) (8 points) Calculate the population mean $\boldsymbol{\mu}$ and standard deviation $\boldsymbol{\sigma}$.

Fallback values for 5(b)–5(d): If you cannot determine $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ in 5(a), you may use $\boldsymbol{\mu} = \mathbf{50}$ micrometers and $\boldsymbol{\sigma} = \mathbf{2.0}$ micrometers for parts 5(b)–5(d). **These are not the correct answers to 5(a)**; they are provided only so you can continue with the rest of the problem. If you use them, say so clearly in your work. You will not receive credit for 5(a) by using these values.

b) (6 points) Find the probability that a randomly chosen wafer has a thickness between 47 micrometers and 53 micrometers.

c) (6 points) Determine the first quartile $\boldsymbol{Q_1}$, the third quartile $\boldsymbol{Q_3}$, and the population Interquartile Range (IQR) for wafer thickness.

d) (6 points) Calculate the population lower and upper inner fences. Using these fences as a quality-control rule, would a manufactured wafer with a thickness of 44.20 micrometers be flagged as a potential outlier requiring further inspection? Explain in a complete sentence.

-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998