

**V1**

Name: _____

PUID _____

Instructor (circle one): Heekyung Ahn Yu Lin Evidence Matangi Timothy Reese Halin Shin

Select Class Meeting Days/Time

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|---|--|
| <input type="radio"/> T/Th 9:00AM-10:15AM | <input type="radio"/> MW 1:30PM-2:45PM |
| <input type="radio"/> MWF 11:30AM-12:20PM | <input type="radio"/> MW 12:30 PM-1:20PM |
| <input type="radio"/> MWF 1:30 PM-2:20PM | <input type="radio"/> MWF 2:30 PM-3:20PM |
| <input type="radio"/> MWF 3:30-4:20PM | <input type="radio"/> MWF 3:30PM-4:20PM <input type="radio"/> Online |

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do.

Accountable together - we are Purdue.

Instructions:

1. Please write your **name** and **PUID** clearly on every **odd page**.
2. **Write your work in the box. Do not run over into the next question space.**
3. The only materials that you are allowed during the exam are your **scientific calculator, writing utensils, erasers, your crib sheet, and your picture ID**. Colored scratch paper will be provided if you need more room for your answers. Please write your name at the top of that paper also.
4. The crib sheet can be a handwritten or typed double-sided 8.5in x 11in sheet.
5. If you share your calculator without permission or have a cell phone at your desk, you will get a **zero** on the exam. Do not take out your cell phone until you are next in line to submit your exam.
6. The exam is only 60 minutes long so there will be no breaks during the exam. If you leave the exam room, you must turn in your exam, and you will not be allowed to come back.
7. **For free response questions you must show ALL your work to obtain full credit.** An answer without showing any work may result in **zero** credit. If your work is not readable, it will be marked wrong. Remember that work must be shown for all numbers that are not provided in the problem or no credit will be given for them. All explanations must be in complete English sentences to receive full credit.
8. All numeric answers should have **four decimal places** unless stated otherwise.
9. After you complete the exam, please turn in your exam as well as your table and any scrap paper that you used. Please be prepared to **show your Purdue picture ID**.
10. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone caught cheating on the exam will automatically fail the course and will be reported to the Office of the Dean of Students.
11. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you on Gradescope after it is graded.

Your exam is not valid without your signature below. This means that it won't be graded.

I attest here that I have read and followed the instructions above honestly while taking this exam and that the work submitted is my own, produced without assistance from books, other people (including other students in this class), notes other than my own crib sheet(s), or other aids. In addition, I agree that if I tell any other student in this class anything about the exam BEFORE they take it, I (and the student that I communicate the information to) will fail the course and be reported to the Office of the Dean of Students for Academic Dishonesty.

Signature of Student: _____

**You may use this page as scratch paper.
The following is for your benefit only.**

Question Number	Total Possible	Your points
Problem 1 (True/False) (2 points each)	12	
Problem 2 (Multiple Choice) (3 points each)	15	
Problem 3	23	
Problem 4	27	
Problem 5	28	
Total	105	

The rest of this page can be used for scratch work

1. (12 points, 2 points each) **True/False Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect.

1.1. Let X and Y be two discrete random variables with supports $x \in \{0, 1, 2\}$, and $y \in \{1, 2, 3\}$.

☐ T or ☒ F If $P(X = 1, Y = 1) = P(X = 1) \cdot P(Y = 1)$, then this implies that X and Y are independent random variables.

1.2. For three events A, B , and C from the same sample space Ω , it is known that $C \neq \emptyset$, $P(A \cap B) > 0$.

☐ T or ☒ F If $P(A \cap B \cap C) = 0$, it must follow that $P(A \cap C) = P(B \cap C) = 0$.

1.3. Let X be a continuous random variable with a **PDF** $f_X(x)$ defined over the symmetric interval $[-c, c]$ for some constant $c > 0$.

☒ T or ☐ F If the PDF is an even function (meaning $f_X(-x) = f_X(x)$) for all x , then this implies that the **50th percentile (median)** of the distribution must be **0**.

1.4. A censor reports a raw error measurement X in millivolts. Historical data show a bias in these measurements with $E[X] = -2$ (millivolts), with $SD(X) = 5$ (millivolts).

☒ T or ☐ F For the transformed score $Y = 5 \cdot X + 35$, it follows that $E[Y] = SD(Y)$.

1.5. In Normal distribution word problems, we distinguish “forward” and “backward” problems.

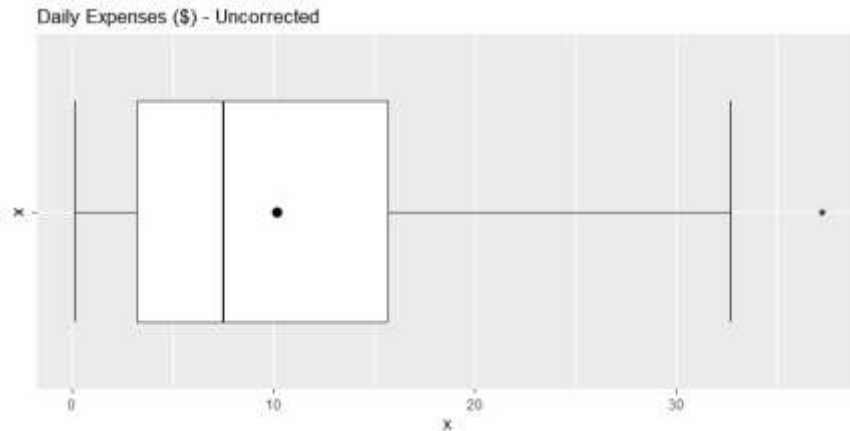
☒ T or ☐ F “Backward” problems solve for an x -value given a probability, while “forward” problems solve for a probability given an x -value.

1.6. In a class of 60 students, the instructor randomly selects 20 homework submissions without replacement to audit for possible AI policy violations. Each audited paper is labeled probable violation or no obvious violation. Let X be the number of audited papers with probable violations.

☐ T or ☒ F Then X is a Binomial random variable because the process satisfies the BINS conditions.

2. (15 points, 3 pts each) **Multiple Choice Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect. **For each question, there is only one correct option letter choice unless specified.**

2.1. A researcher randomly selected 100 graduate students and surveyed their daily expenses on eating out. The collected data are visualized in the boxplot below.



The researcher received an email from one of the participants, stating that they had misreported the amount, and the corrected amount should be **27.24** instead of **37.24**. After this correction, which of the **two graphical components** of the **boxplot** remain **unchanged**?

- ☒ A Third Quartile, Q_3
- ☐ B Sample Mean, \bar{x}
- ☒ C Upper Whisker
- ☐ D The number of explicit points
- ☐ E The maximum value

2.2. At a large clinic, women's heights are well modeled by a Normal distribution with mean 165 cm and standard deviation 7 cm. Identify the **false** statement from A-E.

- ☐ A The mean, median, and mode are all 165 cm.
- ☐ B The 25th and 75th percentiles are equidistant from 165 cm.
- ☐ C Converting heights from centimeters to inches does not change anyone's z-score or percentile.
- ☐ D About 68% of women have heights within 7 cm of 165 cm.
- ☒ E Every woman's height lies within 3 standard deviations of 165 cm.

2.3. A clinical trial enrolls **10 women**, each receiving one dose of a new drug. Based on a pre-measured genotype, **4** participants have a **30%** chance of response (type G_1) and **6** have a **70%** chance (type G_2). Responses are **independent** across participants, and each participant's outcome is either **response** or **no response**. Let X be the number who respond. Identify the **correct** statement from A-E.

- ☐ (A) The number of trials is not a fixed constant.
- ☐ (B) The trials are dependent.
- ☐ (C) There are more than two possible outcomes on each trial.
- ☒ (D) The probability of success is not the same for all trials.
- ☐ (E) The trials are conducted without replacement from a small finite population.

2.4. A new type of biodegradable plastic is developed, and its degradation time, X (in years), is modeled by the probability density function $f_X(x) = kx^2$ for $0 \leq x \leq 3$, and $f_X(x) = 0$ otherwise. What is the expected lifetime of this plastic?

- ☐ (A) 1 years
- ☐ (B) 1.5 years
- ☒ (C) 2.25 years
- ☐ (D) 3 years
- ☐ (E) 3.25 years
- ☐ (F) 9 years

2.5. Let $X \sim \text{Binomial}(n = 2, p)$ with unknown $p \in (0, 1)$. Define a second random variable Y conditionally on X as follows:

- $P(Y = 0|X = 0) = 1$
- $P(Y = 1|X = 1) = P(Y = 2|X = 1) = \frac{1}{2}$
- $P(Y = y|X = 2) \sim \text{Poisson}(\lambda)$

Which expression equals $P(Y = 0)$?

- ☒ (A) $(1 - p)^2 + p^2 \cdot e^{-\lambda}$
- ☐ (B) $(1 - p)^2 \cdot e^{-\lambda} + p^2$
- ☐ (C) $(1 - p)^2 + 2p \cdot (1 - p) + p^2 \cdot e^{-\lambda}$
- ☐ (D) $(1 - p) \cdot p \cdot e^{-\lambda}$

Free Response Questions 3-5. Show all work, clearly label your answers, and use **four decimal places**.

3. **(23 points)** Assume men's college basketball game lengths X (minutes) are Normally distributed with mean 118 and standard deviation 10, where these values already account for fouls, timeouts, media breaks, and overtime.

- a) **(6 points)** Find the probability that a randomly selected game lasts within 1.5 standard deviations of the mean.

$$\begin{aligned}
 X &\sim N(\mu = 118, \sigma = 10) \\
 P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) &= P(-1.5 < Z < 1.5) \\
 &= P(Z < 1.5) - P(Z < -1.5) \\
 &= 2 \cdot P(Z < 1.5) - 1 \\
 &= 2 \cdot 0.9332 - 1 = 0.8664
 \end{aligned}$$

- b) **(10 points)** Compute the inter quartile range (IQR) for the population of men's college basketball game lengths.

For the standard normal the 25th and 75th percentiles are found easily from the table.

$$z_{0.25} = -0.67 \text{ and } z_{0.75} = +0.67$$

Transform to the basketball game length distribution:

$$x_{0.25} = \mu + z_{0.25} \cdot \sigma = 118 - 0.67 \cdot 10 = 111.3$$

$$x_{0.75} = \mu + z_{0.75} \cdot \sigma = 118 + 0.67 \cdot 10 = 124.7$$

$$\text{IQR} = x_{0.75} - x_{0.25} = 124.7 - 111.3 = 13.4$$

- c) **(4 points)** A random sample of 10 game times is given below. Compute the population level inner fences and determine if any of these points fall outside the 1.5 IQR rule.

88 89 108 111 115 115 116 124 130 134

$$\text{Inner lower fence: } x_{0.25} - 1.5 \cdot \text{IQR} = 111.3 - 1.5 \cdot 13.4 = 91.2$$

$$\text{Inner upper fence: } x_{0.75} + 1.5 \cdot \text{IQR} = 124.7 + 1.5 \cdot 13.4 = 144.8$$

We have two points that fall outside the inner lower fence and those points are 88 and 89.

d) (3 points) Consider three distribution models used in this course: **Normal**(μ, σ^2), **Exponential**(λ), and **Uniform**(a, b). The interquartile range ($IQR = Q_3 - Q_1$) measures the spread of the middle 50% of the distribution. In the statements below, “does not depend on the mean” means the IQR cannot be determined from the mean ($E[X]$) alone; “constant multiple of the mean” means $IQR = k \cdot E[X]$ for some constant k that does not vary. (If needed use scratch space on pg 2 of exam.) Which statement about the **IQR** and the mean ($E[X]$) is **incorrect**?

- Ⓐ For an **Exponential** distribution, the **IQR** is a constant multiple of the mean.
- Ⓑ For a **Normal** distribution, the **IQR** does not depend on the mean.
- Ⓒ For a **Uniform** distribution on $(0, b)$, the **IQR** equals the mean.
- Ⓓ For a **Uniform** distribution on (a, b) , the **IQR** is determined by the mean alone.

4. (27 points) Heekyung is training her cat, Meredith, to give a high-five. From her extensive experience, Meredith’s response depends on whether a treat is offered. Heekyung **offers a treat 70% of the time**. Meredith’s behavior on an attempt is exactly one of the following: high-five, ignore, or nag.

If a treat is offered:

- Meredith gives a high-five with probability 0.8.
- Meredith ignores Heekyung with probability 0.2.

If a treat is not offered:

- Meredith gives a high-five with probability 0.35.
- Meredith ignores Heekyung with probability 0.6.
- Otherwise, Meredith Nags Heekyung.

Heekyung will restart the training next month if the probability that no treat was offered, given that a high-five occurred, is greater than one-half.

a) (3 points) Are the two events **{Treat is offered}** and **{Give a high-five}** independent? State yes or no and provide mathematical justification.

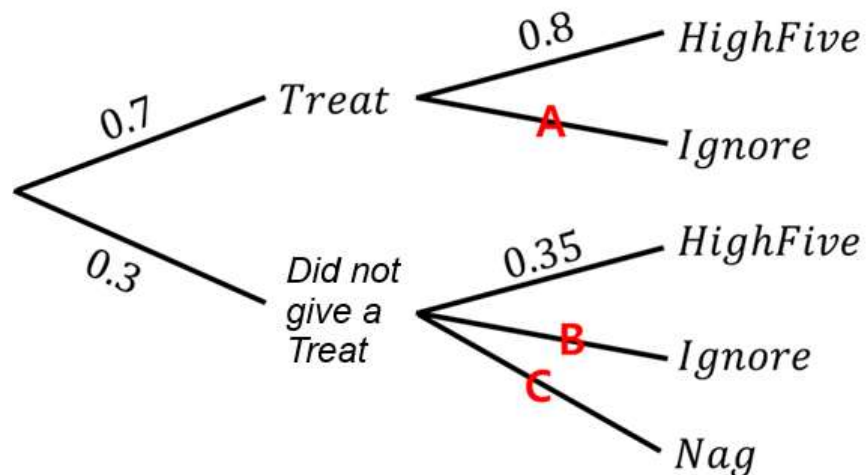
The events are not independent.

$P(\{\text{Give a high five}\}|\{\text{Treat is offered}\})$

$\neq P(\{\text{Give a high five}\}|\{\text{Treat is not offered}\})$

which in turn means the probability that Meredith gives a high five is dependent on whether Meredith is offered a treat or not.

- b) (2 points) For each branch A, B, and C in the tree diagram below, write the probability statement and find its probability.



$$A \rightarrow P(\{\text{Ignore}\}|\{\text{Treat is offered}\}) = 1 - 0.8 = 0.2$$

$$B \rightarrow P(\{\text{Ignore}\}|\{\text{Treat is not offered}\}) = 0.6$$

$$C \rightarrow P(\{\text{Nag}\}|\{\text{Treat is not offered}\}) = 1 - 0.35 - 0.6 = 0.05$$

- c) (4 points) Determine the probability that Meredith Nags Heekyung.

Law of total probability:

$$\begin{aligned} P(\{\text{Nag}\}) &= P(\{\text{Nag}\}|\{\text{Treat is offered}\}) \cdot P(\{\text{Treat is offered}\}) \\ &\quad + P(\{\text{Nag}\}|\{\text{Treat is not offered}\}) \cdot P(\{\text{Treat is not offered}\}) \\ &= 0 \cdot 0.7 + 0.05 \cdot 0.3 = 0.015 \end{aligned}$$

d) (8 points) Determine the probability that Meredith gives a high-five.

Law of total probability:

$$\begin{aligned}
 P(\{\text{Give a high five}\}) &= P(\{\text{Give a high five}\}|\{\text{Treat is offered}\}) \cdot P(\{\text{Treat is offered}\}) \\
 &+ P(\{\text{Give a high five}\}|\{\text{Treat is not offered}\}) \cdot P(\{\text{Treat is not offered}\}) \\
 &= 0.8 \cdot 0.7 + 0.35 \cdot 0.3 = 0.665
 \end{aligned}$$

e) (10 points) Based on your results, compute the probability that no treat was offered, given that a high-five occurred. According to these calculations and Heekyung's initial no retraining requirements stated at the start of this question, should Heekyung restart the training next month?

Bayes Rule:

$$\begin{aligned}
 P(\{\text{Treat is not offered}\}|\{\text{Give a high five}\}) &= \frac{P(\{\text{Give a high five}\}|\{\text{Treat is not offered}\}) \cdot P(\{\text{Treat is not offered}\})}{P(\{\text{Give a high five}\})} \\
 &= \frac{0.35 \cdot 0.3}{0.665} \approx 0.1579
 \end{aligned}$$

Do not restart training as the probability is less than 0.5. (Did not grade this part as we should have said will not restart training if probability is greater than 0.5)

5. (28 points) Purdue analyzed how long tickets take to resolve. Routine requests are handled in a quick “triage window” and are about equally likely to finish at any time during the **first 10 minutes**. If a ticket survives past 10 minutes, the remaining time decays exponentially. Let T be the resolution time (minutes) with

$$f_T(t) = \begin{cases} k & 0 \leq t \leq 10 \\ ke^{-(t-10)} & t > 10 \\ 0 & \text{otherwise} \end{cases}$$

- a) (10 points) Determine k so that f_T is a valid probability density function.

$$k > 0 \text{ otherwise } f_T(t) < 0$$

Solve for k such that

$$\begin{aligned} \int_{-\infty}^{\infty} f_T(t) dt &= 1 \\ \int_0^{10} k dt + \int_{10}^{\infty} ke^{-(t-10)} dt &= 1 \\ 10 \cdot k + k &= 1 \\ k &= \frac{1}{11} \end{aligned}$$

- b) (6 points) What is the probability that a ticket is finished between 5 and 15 minutes? For your convenience, the CDF of the resolution time T is given below, defined up to the normalizing constant k determined in part a).

$$F_T(t) = \begin{cases} 0 & t < 0 \\ k \cdot t & 0 \leq t \leq 10 \\ 1 - k \cdot e^{-(t-10)} & t > 10 \end{cases}$$

$$\begin{aligned} F_T(t) &= \begin{cases} 0 & t < 0 \\ \frac{t}{11} & 0 \leq t \leq 10 \\ 1 - \frac{1}{11} \cdot e^{-(t-10)} & t > 10 \end{cases} \\ P(5 < T < 15) &= F_T(15) - F_T(5) \\ &= 1 - \frac{1}{11}e^{-5} - \frac{5}{11} \approx 0.5448 \end{aligned}$$

- c) (6 points) Given that a ticket has been in the system for at least **5 minutes**, determine the probability that the total time the ticket remains unresolved is **at least 15 minutes**.

$$P(T \geq 15 | T \geq 5) = \frac{1 - F_T(15)}{1 - F_T(5)} = \frac{1 - \left(1 - \frac{1}{11}e^{-5}\right)}{1 - \frac{5}{11}} = \frac{e^{-5}}{6} \approx 0.0011$$

- d) (6 points) Only **5%** of tickets take longer than t^* to solve. Determine t^* .

Solve for t^* such that $P(T > t^*) = 0.05$, t^* must fall into region $t > 10$ because $F_T(10) = \frac{10}{11}$ which is less than 0.95.

$$1 - F_T(t^*) = 0.05$$

$$\frac{e^{-(t^*-10)}}{11} = 0.05$$

$$-(t^* - 10) = \ln(0.55)$$

$$t^* = 10 - \ln(0.55)$$

$$t^* \approx 10.5978$$

You may use this page as scratch paper

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

