STAT 350 Exam 1 SPRING 2024

V1

Name: ______________________________

PUID ___________________________

Instructor (circle one): Anand Dixit Timothy Reese Halin Shin Heekyung Ahn

Class Start Time: ○ **9:30 AM** ○ **11:30 PM** ○ **1:30 PM** ○ **2:30 PM** ○ **3:30 PM** ○ **Online**

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

Instructions:

1. **IMPORTANT** Please write your **name** and **PUID clearly** on every **odd page.**
2. **Write your work in the box. Do not run over into the next question space.**
3. You are expected to uphold the honor code of Purdue University. It is your responsibility to keep your work covered at all times. Anyone caught cheating on the exam will automatically fail the course and will be reported to the Office of the Dean of Students.
4. It is strictly prohibited to smuggle this exam outside. Your exam will be returned to you on Gradescope after it is graded.
5. The only materials that you are allowed during the exam are your **scientific calculator**, **writing utensils, erasers, your crib sheet**, and **your picture ID**. If you bring any other papers into the exam, you will get a **zero** on the exam. Colored scratch paper will be provided if you need more room for your answers. Please write your name at the top of that paper also.
6. The crib sheet can be a handwritten or type double-sided 8.5in x 11in sheet.
7. Keep your bag closed and cellphone stored away securely at all times during the exam.
8. If you share your calculator or have a cell phone at your desk, you will get a **zero** on the exam.
9. The exam is only 60 minutes long so there will be no breaks (including bathroom breaks) during the exam. If you leave the exam room, you must turn in your exam, and you will not be allowed to come back.
10. **For free response questions you must show ALL your work to obtain full credit.** An answer without showing any work may result in **zero** credit. If your work is not readable, it will be marked wrong. Remember that work has to be shown for all numbers that are not provided in the problem or no credit will be given for them. All explanations must be in complete English sentences to receive full credit.
11. All numeric answers should have **four decimal places** unless stated otherwise.
12. After you complete the exam, please turn in your exam as well as your table and any scrap paper that you used. Please be prepared to **show your Purdue picture ID**. You will need to **sign a sheet** indicating that you have turned in your exam.

Your exam is not valid without your signature below. This means that it won't be graded.

I attest here that I have read and followed the instructions above honestly while taking this exam and that the work submitted is my own, produced without assistance from books, other people (including other students in this class), notes other than my own crib sheet(s), or other aids. In addition, I agree that if I tell any other student in this class anything about the exam BEFORE they take it, I (and the student that I communicate the information to) will fail the course and be reported to the Office of the Dean of Students for Academic Dishonesty.

Signature of Student: ____

You may use this page as scratch paper.
The following is for your benefit only.

Question Number	Total Possible	Your points
Problem 1 (True/False) (2 points each)	12	
Problem 2 (Multiple Choice) (3 points each)	15	
Problem 3	24	
Problem 4	24	
Problem 5	30	
Total	105	

1. **(12 points, 2 points each) True/False Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect.

1.1. Let $\boldsymbol{X} \sim \mathbf{Binomial}(\boldsymbol{n}, \boldsymbol{p} = \mathbf{0.5})$, where $\boldsymbol{n}$ is any positive integer.

Ⓣ or Ⓕ For any value of $\boldsymbol{x}$ in the support of $\boldsymbol{X}$, $\boldsymbol{P}(\boldsymbol{X} = \boldsymbol{x}) = \boldsymbol{P}(\boldsymbol{X} = \boldsymbol{n} - \boldsymbol{x})$.

1.2. Suppose two events $\boldsymbol{A}$ and $\boldsymbol{B}$ are in the sample space $\boldsymbol{\Omega}$ with all outcomes of $\boldsymbol{A}$ contained within the event $\boldsymbol{B}$.

Ⓣ or Ⓕ In this scenario it must follow that $\boldsymbol{P}(\boldsymbol{A} \cap \boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{B})$.

1.3. Given two non-empty events $\boldsymbol{A}$ and $\boldsymbol{B}$ of a sample space $\boldsymbol{\Omega}$,

Ⓣ or Ⓕ if $\boldsymbol{P}(\boldsymbol{A}|\boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{A})$ then we are certain that $\boldsymbol{A} \cap \boldsymbol{B} \neq \emptyset$.

1.4. Let $\boldsymbol{X}$ be a random variable that satisfies the conditions to be distributed as Poisson.

Ⓣ or Ⓕ The expected value must satisfy $\mathbf{E}[\boldsymbol{X}] > \mathbf{0}$.

1.5. Given a five number summary for a dataset we could compute the interquartile range, identify the fences, and draw a modified box plot to visualize properties of the data.

Ⓣ or Ⓕ The upper whisker of the modified boxplot would be drawn to terminate at the point $\boldsymbol{Q}_3 + \mathbf{1.5} \times \mathbf{IQR}$.

1.6. For a random variable $\boldsymbol{X}$ that follows a normal distribution,

Ⓣ or Ⓕ the mode of $\boldsymbol{X}$ is always greater than its mean.

2. **(15 points, 3 points each) Multiple Choice Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect. **For each question, there is only one correct option letter choice.**

2.1. Let $\boldsymbol{X}$ be a random variable with mean $\boldsymbol{\mu_X = 7}$ and standard deviation $\boldsymbol{\sigma_X = 9}$. Define another random variable $\boldsymbol{Y = 2X^2 + 5X + 3}$. Determine the value of $\boldsymbol{E[Y]}$.

Ⓐ $\mathbf{E}[Y] = \mathbf{136}$

Ⓑ $\mathbf{E}[Y] = \mathbf{200}$

Ⓒ $\mathbf{E}[Y] = \mathbf{214}$

Ⓓ $\mathbf{E}[Y] = \mathbf{298}$

Ⓔ Not enough information to calculate it.

2.2. Identify the false statement regarding a continuous random variable $\boldsymbol{Y}$, which has support extending from 0 to infinity:
(Note: The pdf and cdf of the random variable $\boldsymbol{Y}$ is denoted by $\boldsymbol{f_Y(y)}$ and $\boldsymbol{F_Y(y)}$ respectively.)

Ⓐ If $\boldsymbol{y_1}$and $\boldsymbol{y_2}$ are values in the support with $\boldsymbol{y_1 < y_2}$, then it follows that $\boldsymbol{F_Y(y_1) \leq F_Y(y_2)}$.

Ⓑ If $\boldsymbol{y}$ is a value in the support, $\boldsymbol{f_Y(y) > 0}$.

Ⓒ If $\boldsymbol{y}$ is a value in the support, $\boldsymbol{P(Y = y) > 0}$.

Ⓓ It is possible for $\boldsymbol{f_Y(y)}$ to be a decreasing function for all values of $\boldsymbol{y}$ in the support.

Ⓔ If $\boldsymbol{y_1}$and $\boldsymbol{y_2}$ are values in the support with $\boldsymbol{y_1 < y_2}$, it is possible that $\boldsymbol{f_Y(y_1) > f_Y(y_2)}$.

2.3. In Cerulean city, 3.2 car accidents are reported on average per day. The number of car accidents is known to follow a Poisson distribution. What is the probability that at least one accident occurs per day? Let $\boldsymbol{X}$ denote the Poisson random variable for this situation.

Ⓐ $\boldsymbol{P(X \geq 1) = 0.9592}$

Ⓑ $\boldsymbol{P(X \geq 1) = 0.0408}$

Ⓒ $\boldsymbol{P(X \geq 1) = 0.1712}$

Ⓓ $\boldsymbol{P(X \geq 1) = 0.8288}$

Ⓔ None of the above

2.4. Suppose a random variable $\boldsymbol{X}$ follows a normal distribution with an unknown mean, **μ**, and unknown variance, $\boldsymbol{\sigma^2}$. Then, $\boldsymbol{P(X > \mu + \sigma)}$ is approximately equal to?

Ⓐ **0.16**

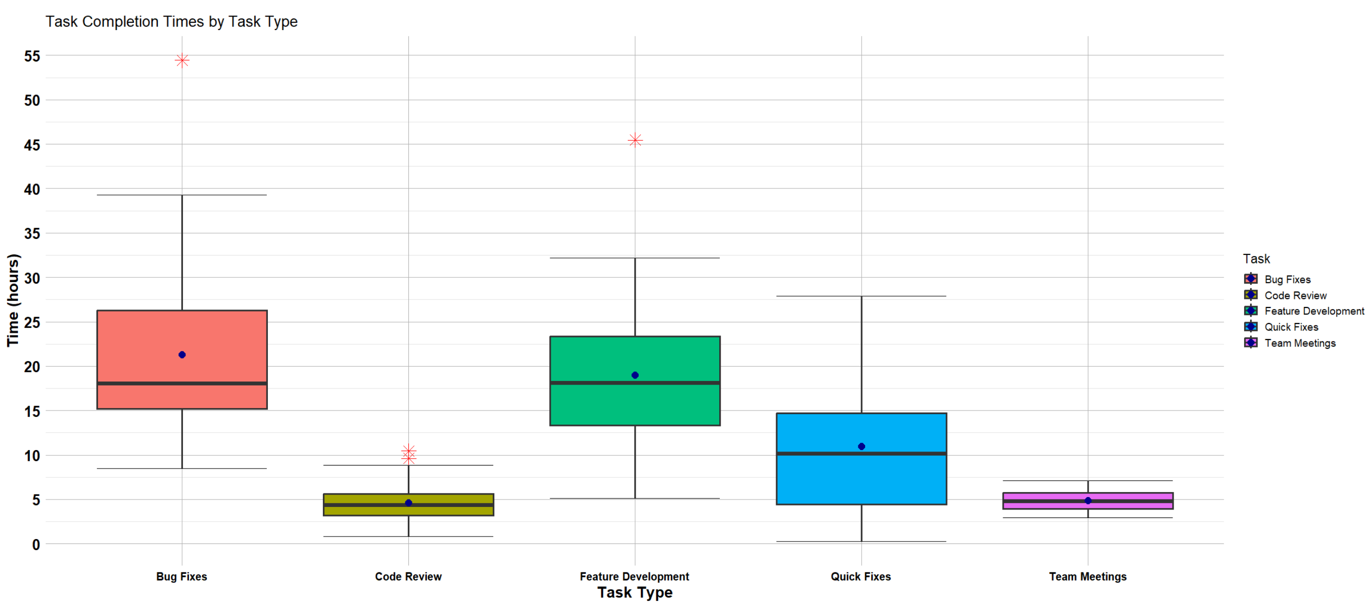
Ⓑ **0**

Ⓒ **0.25**

Ⓓ **0.5**

Ⓔ Not enough information available.

2.5. At Lumina Tech, a software development company, the project management team conducted an analysis to understand the distribution of time spent on five categories: Quick Fixes, Feature Development, Bug Fixes, Code Review, and Team Meetings. Analyzing monthly data of 32 employees, they generated side-by-side boxplots for each category to illustrate time distributions. Using the side-by-side boxplot approximate the number of employees that spent more than 15 hours on **Bug Fixes**.



Ⓐ **8 employees**

Ⓑ **16 employees**

Ⓒ **24 employees**

Ⓓ **30 employees**

Ⓔ Not enough information available.

Free Response Questions 3-5. Show all work, clearly label your answers, and use **four decimal places**.

3. **(24 points)** Marina orders her dinner from Doordash. When she places an order, it is known to take 35 minutes on average for delivery. Assume each order's delivery time is independent of others.

   a) **(4 points)** Define the continuous random variable $\boldsymbol{X}$ which represents the amount of time (in minutes) Marina waits for her delivery. Write the name of its distribution and provide the value of the parameter, $\boldsymbol{\lambda}$ or $\boldsymbol{\mu}$.

$$X \sim \text{Exponential}\left(\lambda = \frac{1}{35}\right)$$

The pdf is $f_X(x) = \frac{1}{35} e^{-\frac{x}{35}}$ for $x \geq 0$

   b) **(4 points)** What is the probability that Marina will wait exactly 38 minutes for her delivery?

Since this is a continuous random variable the probability of being exactly 38 is 0.

$$P(X = 38) = 0$$

   c) **(6 points)** What is the probability that Marina will wait more than 25 minutes for her delivery?

$$P(X > 25) = \int_{25}^{\infty} \frac{1}{35} e^{-\frac{x}{35}}\, dx = e^{-\frac{25}{35}} = 0.4895$$

   d) **(10 points)** If the delivery takes less than 25 minutes, Marina will add an additional tip to the deliverer. Assume she placed 10 orders in January. What is the probability that Marina adds additional tip for 2 orders out of 10 orders?

Let Y denote a new random variable that counts the number of orders that marina adds an additional tip. This is a Binomial random experiment.

$$Y \sim \text{Bin}\left(n = 10, p = 1 - e^{-\frac{25}{35}}\right)$$

$$P(Y = 2) = \binom{10}{2}\left(1 - e^{-\frac{25}{35}}\right)^2 \left(e^{-\frac{25}{35}}\right)^8 = 0.0387$$

4. (24 points) Health authorities at Lumina University observed that the weights of their students follow a normal distribution. Furthermore, their assessment revealed that the mean weight of their students is 160 lbs, with a variance of 25 lbs^2. Using this information, answer the following questions:

a) **(6 points)** What is the probability that a student at Lumina University weighs at least 156 lbs?

Let W be defined as the weight of a random student at Lumina University.

$$W \sim \text{Normal}(\mu = 160, \sigma^2 = 25)$$

$$P(W \geq 156) = P\left(Z \geq \frac{156 - 160}{5}\right) = P(Z \geq -0.8) = P(Z < 0.8) = 0.7881$$

b) **(8 points)** What is the probability that a student at Lumina University weighs between 160 lbs and 168 lbs?

$$P(160 < W < 168) = P(W < 168) - P(W < 160)$$

$$= P\left(Z < \frac{168 - 160}{5}\right) - P\left(Z < \frac{160 - 160}{5}\right)$$

$$= P(Z < 1.6) - P(Z < 0) = 0.9452 - 0.5 = 0.4452$$

c) **(10 points)** Lumina University's health officials declared 0.25% of students overweight. What cutoff value was used by them to determine whether a student is overweight or not?

The upper 0.25% is the same cutoff as the lower 99.75% percentile.

$$P(Z < z_{0.9975}) = 0.9975$$

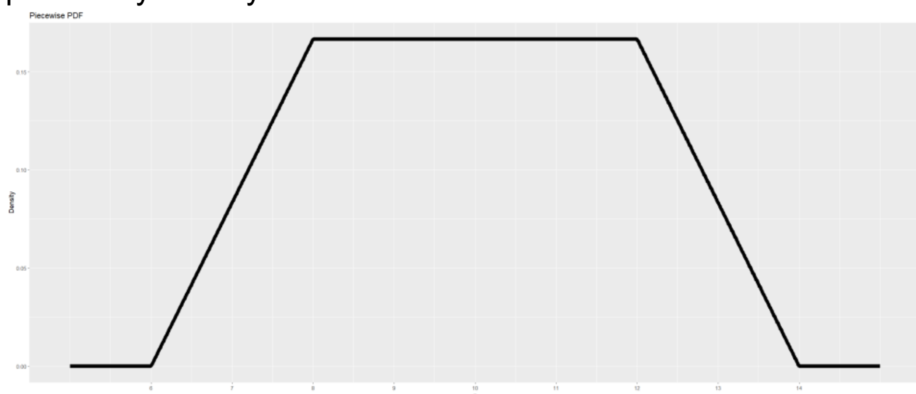
$$\rightarrow z_{0.9975} = 2.81$$

$$w_{0.9975} = 160 + 2.81 * 5 = 174.05$$

5. **(30 points)** In a nanotechnology lab, researchers study ultraviolet light's effect on nanostructures, focusing on wavelengths between 6 and 14 nanometers. The probability density function (PDF) models the distribution of these interactions, aiding in the development of materials with specific optical properties. Before analyzing these interactions, it's crucial to establish the normalizing constant for precise probability calculations.

$$f_X(x) = \begin{cases} k\,(x-6) & 6 \le x \le 8 \\ 2\text{k} & 8 < x \le 12 \\ \text{k}\,(14-x) & 12 < x \le 14 \\ 0 & \text{otherwise} \end{cases}$$

a) **(10 points)** Determine the value of the constant k such that the function is a valid probability density function.



First, we sketch out $f_X(x)$ to see that it is always non-negative as long as $k > 0$.

Solve:

$$k\left[\int_6^8 (x-6)dx + 2\int_8^{12} dx + \int_{12}^{14} (14-x)dx\right] = 1$$

Quick u-substitution: $u = x - 6$ **and** $v = 14 - x$.

$$k\left[\int_0^2 udu + 2\int_8^{12} dx - \int_2^0 vdv\right] = 1$$

$$k\left[\frac{u^2}{2}\Big|_0^2 + 2 \times (12-8) + \frac{v^2}{2}\Big|_0^2\right] = 1$$

$$k[2+8+2\,] = 1$$

$$k = \frac{1}{12}$$

$$F_X(x) = \begin{cases} [A] & x < 6 \\ [B] & 6 \leq x < 8 \\ \dfrac{x-7}{6} & 8 \leq x < 12 \\ 1 - \dfrac{(x-14)^2}{24} & 12 \leq x < 14 \\ [C] & x \geq 14 \end{cases}$$

b) **(6 points)** Determine the missing parts **[A]**, **[B]**, and **[C]** for the cumulative distribution function $F_X(x)$ above.

$[A] = 0$ (Have not reached support yet no area)

$[C] = 1$ (Accumulated the entire area after passing 14)

$[B] = \frac{1}{12}\int_6^x (t-6)dt = \frac{(x-6)^2}{24}$

c) **(6 points)** Determine the probability that the wavelength of UV light interacting with a nanostructure is between 10 and 12 nanometers.

$$P(10 < X < 12) = F_X(12) - F_X(10)$$
$$= \frac{5}{6} - \frac{3}{6} = \frac{1}{3} = 0.3333$$

d) **(4 points)** Calculate the mean wavelength (expected value) of UV light interacting with nanostructures.

The distribution is symmetric about 10 and therefore the mean and median are both 10.

$\boldsymbol{E[X] = 10}$

e) **(4 points)** The function $\boldsymbol{g(X) = 0.1X - 0.5}$ approximates the intensity of light absorption of the nanostructures. Given that the variance of the UV light interacting with the nanostructures is known to be $\boldsymbol{\sigma_X^2 = \frac{10}{3}}$, determine the standard deviation of the intensity of light absorption of the nanostructures.

$$\sqrt{Var(g(X))} = \sqrt{Var(0.1X - 0.5)} = \sqrt{0.1^2 Var(X)} = 0.1 \times \sqrt{10/3} = 0.1826$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998