STAT 350 Exam 2 SPRING 2026

V1

Name: ___________________________________

PUID ____________________________

Instructor (circle one):

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Select Class Meeting Days/Time

○ **MWF** 10:30AM-11:20AM ○ **MWF** 2:30PM-3:20PM
○ **MWF** 11:30AM-12:20PM ○ **MWF** 3:30PM-4:20PM
○ **MWF** 12:30PM-1:20PM ○ **T/TH** 1:30PM-2:45PM
○ **MWF** 1:30PM-2:20PM ○ **T/TH** 3:00PM-4:15PM ○ **Online**

Exam Instructions

1. **Identification:** Write your last name clearly on every odd page and on any provided scratch paper. Have your Purdue picture ID ready when submitting the exam.
2. **Allowed Materials:** Scientific calculator, writing utensils, erasers, and one double-sided 8.5" x 11" crib sheet (handwritten or typed).
3. **Formatting & Work:** Keep all work strictly within the designated boxes. Round all numeric answers to **four decimal places** unless stated otherwise.
4. **Grading Requirements:** You must show ALL work for free-response questions. Unsupported numbers, unreadable work, or missing explanations (which must be in complete English sentences) will receive zero credit.
5. **Logistics:** The exam is strictly 60 minutes with no breaks. Leaving the room finalizes your submission. Submit your exam, tables, and scratch paper together. Exams will be returned via Gradescope.

Strict Prohibitions & Honor Code

- **Zero Tolerance:** Absolutely no cell phones at your desk. No sharing calculators. Violating either rule results in an immediate zero on the exam.
- **Academic Integrity:** You are responsible for keeping your work covered at all times. The unauthorized removal of exam materials from the room, any form of academic dishonesty, or discussing exam contents with students who have not yet tested will result in an **automatic course failure** and a formal report to the Office of the Dean of Students.

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

Your exam is not valid without your signature below. This means that it won't be graded.
I attest that I have read and followed the instructions above honestly. The work submitted is my own, produced without unauthorized assistance. I agree that if I share information about this exam with any student before they take it, both parties will fail the course and be reported for Academic Dishonesty.

Signature of Student: ____

You may use this page as scratch paper.
The following is for your benefit only.

Question Number	Total Possible	Your points
Problem 1 (True/False) (2 points each)	12	
Problem 2 (Multiple Choice) (3 points each)	15	
Problem 3	26	
Problem 4	26	
Problem 5	26	
Total	105	

The rest of this page can be used for scratch work

1. **(12 points, 2 points each) True/False Questions.** Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect.

   1.1. An automotive engineer is studying how tire compound (soft, medium, hard) and suspension stiffness (low, high) affect braking distance. Because vehicle weight is known to influence braking performance, the engineer groups 30 test vehicles into five blocks based on weight class (subcompact, compact, midsize, full-size, SUV). Within each weight class, the 6 vehicles are randomly assigned so that each of the six treatment combinations is applied to exactly one vehicle. The braking distance (in meters) from 100 km/h is recorded.

   Ⓣ or Ⓕ In this randomized block design, the blocks are defined by all combinations of tire compound, suspension stiffness, and vehicle weight class, resulting in 30 total blocks.

   1.2. In the context of hypothesis testing and confidence intervals, let $\boldsymbol{C}$ denote the confidence level of an interval, $\boldsymbol{\alpha}$ denote the complementary significance level (so that ($\boldsymbol{C} + \boldsymbol{\alpha} = \mathbf{1}$), and $\boldsymbol{\beta}$ denote the probability of a Type II error.

   Ⓣ or Ⓕ Since $\boldsymbol{C} + \boldsymbol{\alpha} = \mathbf{1}$ and power $+ \boldsymbol{\beta} = \mathbf{1}$, it follows that $\boldsymbol{C} =$ power.

   1.3. An aerospace engineer constructs a confidence interval for the mean drag coefficient $\boldsymbol{\mu}$ of a new wing design from a random sample of fixed size $\boldsymbol{n}$.

   Ⓣ or Ⓕ The confidence level $\boldsymbol{C}$ is one of the factors that determines the width of the confidence interval.

   1.4. A quality engineer is planning a hypothesis test to detect whether the mean diameter of manufactured bolts has shifted from the specification value $\boldsymbol{\mu_0} = \mathbf{10.00}$ mm. She is evaluating the statistical power of the test at a specific alternative value $\boldsymbol{\mu_a}$ that belongs to the alternative hypothesis.

   Ⓣ or Ⓕ The statistical power of a hypothesis test may decrease when the sample size $\boldsymbol{n}$ is increased.

   1.5. A mechanical engineer tests the fatigue life of $\boldsymbol{n} = \mathbf{40}$ aluminum alloy specimens and constructs a 95% confidence interval for the true mean fatigue life $\boldsymbol{\mu}$ (in cycles). The computed interval is $(\mathbf{22.5},\ \mathbf{22.7})$.

   Ⓣ or Ⓕ It is incorrect to say that $\mathbf{22.5} \le \boldsymbol{\mu} \le \mathbf{22.7}$ with 0.95 probability, since the inequality does not involve any random variables.

   1.6. A researcher wishes to test $\boldsymbol{H_0}: \boldsymbol{\mu} \le \boldsymbol{\mu_0}$ versus $\boldsymbol{H_a}: \boldsymbol{\mu} > \boldsymbol{\mu_0}$ at significance level $\boldsymbol{\alpha} = \mathbf{0.01}$. Before conducting the one-sided test, she constructs a 99% two-sided confidence interval for $\boldsymbol{\mu}$ from the same data and observes that $\boldsymbol{\mu_0}$ falls inside the interval.

   Ⓣ or Ⓕ The researcher can conclude that the one-sided test at $\boldsymbol{\alpha} = \mathbf{0.01}$ will fail to reject $\mathbf{H_0}$.

2. (15 points, 3 pts each) Multiple Choice Questions. Indicate the correct answer by completely filling in the appropriate circle. If you indicate your answer by any other way, you may be marked incorrect. **For each question, there is only one correct option letter choice unless specified.**

2.1. A random sample of size n is drawn from a population with mean μ and finite standard deviation σ. The population distribution is **heavily right-skewed**. Which of the following statements is **FALSE**?

Ⓐ Regardless of sample size, $E[\bar{X}] = \mu$.

Ⓑ For sufficiently large n, the sampling distribution of $\bar{X}$ is approximately normal.

Ⓒ As the sample size n becomes sufficiently large, the observations $X_1, X_2, \dots, X_n$ become approximately normal.

Ⓓ The standard deviation of $\bar{X}$ decreases as n increases, regardless of whether $\bar{X}$ is approximately normal.

Ⓔ A larger sample size n is needed for the CLT to provide a good approximation here than would be needed if the population were only mildly skewed.

2.2. A Purdue scouting intern for the Indianapolis Colts is evaluating 40-yard dash times from $n = 9$ prospective NFL running backs at the 2026 combine. Historical records indicate that 40-yard dash times for running backs are normally distributed with a known population standard deviation of $\sigma = 0.0853$ seconds. The intern's sample yields $\bar{x} = 4.4639$ seconds. To establish the slowest acceptable mean sprint time for recruitment, the intern constructs a **one-sided upper confidence bound** at $\alpha = 0.03$. Which **R** expression gives the correct critical value?

Ⓐ `qnorm(0.03, lower.tail = FALSE)`

Ⓑ `qt(0.03, df = 8, lower.tail = FALSE)`

Ⓒ `qnorm(0.03/2, lower.tail = FALSE)`

Ⓓ `qt(0.03/2, df = 8, lower.tail = FALSE)`

Ⓔ None of the above

2.3. A biomedical engineer wants to compare three physical therapy protocols for post-surgical knee recovery. She recruits 60 patients and knows from prior studies that age strongly affects recovery outcomes. She divides patients into four age groups (18–30, 31–45, 46–60, 61+), and within each age group, randomly assigns an equal number of patients to each of the three protocols. Recovery is measured by range of motion (in degrees) at 8 weeks. Which of the following statements is **FALSE**?

Ⓐ The experimental design is a Randomized Block Design, with age group as the blocking variable.

Ⓑ There are three treatments and four blocks in this experiment.

Ⓒ The purpose of blocking by age is to reduce the variability arising from age differences, making it easier to detect treatment effects.

Ⓓ The blocking ensures that the randomization of patients to protocols is no longer necessary within each age group.

Ⓔ If age had no effect on recovery, a Completely Randomized Design would have been equally effective.

2.4. The table below summarizes the properties of two independent populations **A** and **B**.

	Population A	**Population B**
Distribution family	Normal	
Mean	$\mu_A = \mu_B = \mu$	
Standard Deviation	$\sigma_A = 4.5$	$\sigma_B = 3.9$
Sample size	$n_A = 44$	$n_B = 52$

Which of the following statements about the sampling distributions of $\bar{X}_A$ and $\bar{X}_B$ is **FALSE**?

Ⓐ $P(\bar{X}_A = 4.5) = P(\bar{X}_B = 3.9)$

Ⓑ The pdf of $\bar{X}_A$ and $\bar{X}_B$ have the same value when evaluated at μ. That is, $f_{\bar{X}_A}(\mu) = f_{\bar{X}_B}(\mu)$.

Ⓒ $P(\bar{X}_A \leq \mu - 1) > P(\bar{X}_B \leq \mu - 1)$

Ⓓ $P\left(\bar{X}_A > \mu + \frac{4.5}{\sqrt{44}}\right) = P\left(\bar{X}_B > \mu + \frac{3.9}{\sqrt{52}}\right)$

2.5. An industrial engineer tests whether a process improvement has increased the mean production rate above the current standard of $\boldsymbol{\mu_0 = 200}$ units/hour. The population standard deviation is known to be $\boldsymbol{\sigma = 18}$ units/hour. She collects a sample of $\boldsymbol{n = 36}$ observations and conducts an upper-tailed $\boldsymbol{z}$**-test** at $\boldsymbol{\alpha = 0.05}$. The engineer wants to know the probability that this test will correctly detect an increase if the true mean has shifted to $\boldsymbol{\mu_a = 206}$ units/hour.

The following R outputs are provided below to assist your calculations.

```
> qnorm(0.05, lower.tail = FALSE)
[1] 1.644854

> qnorm(0.025, lower.tail = FALSE)
[1] 1.959964
```

Ⓐ `pnorm(204.9346, mean = 206, sd = 3, lower.tail = FALSE)`

Ⓑ `pnorm(204.9346, mean = 206, sd = 18, lower.tail = FALSE)`

Ⓒ `pnorm(205.8799, mean = 206, sd = 3, lower.tail = FALSE)`

Ⓓ `pnorm(204.9346, mean = 200, sd = 3, lower.tail = FALSE)`

Ⓔ `pnorm(204.9346, mean = 206, sd = 3, lower.tail = TRUE)`

Free Response Questions 3-5. Show all work, clearly label your answers, and use **four decimal places**.

3. **(26 points)** PineApple is assessing the battery life consistency of their next-generation AirBuds. It is known that the battery life of a new left AirBud follows a fairly symmetric distribution with a true mean operation time of $\boldsymbol{\mu_{\mathrm{L}}} = \mathbf{540}$ minutes and a standard deviation of $\boldsymbol{\sigma_{\mathrm{L}}} = \mathbf{81}$ minutes.
   The company will randomly select a batch of $\boldsymbol{n_{\mathrm{L}}} = \mathbf{53}$ left AirBuds from their manufacturing lines and record their battery lifetimes while playing the same audio on repeat. Let $\overline{\boldsymbol{X}}_{\mathbf{L}}$ denote the random variable representing the average battery life of such a randomly selected batch.

   a) **(5 points)** State the approximate distribution of $\overline{\boldsymbol{X}}_{\mathbf{L}}$. Include the name of its distribution family and its parameters (mean and standard error) in both symbolic and numerical forms.

$$\overline{X}_{\mathrm{L}} \approx \mathbf{Normal}\left(\mu_{\mathrm{L}} = 540, \sigma_{\overline{X}_{\mathrm{L}}} = \frac{81}{\sqrt{53}}\right)$$

   b) **(3 points)** What important theorem justifies your approximation in Part (a)?

The Central Limit Theorem (CLT). The original population was reasonably symmetric and $\boldsymbol{n_{\mathrm{L}}} = \mathbf{53}$ is of reasonable size.

The following **R** calculations are provided for parts c) and d). For each of the R commands below the default behavior is `(lower.tail = TRUE)`. You must show all steps clearly.

```
> pnorm(-2.2469)
[1] 0.0123
> pnorm(-2.1424)
[1] 0.0161
```

```
> pnorm(-1.8363)
[1] 0.0332
> pnorm(-1.7976)
[1] 0.0361
```

```
> pnorm(0.6121)
[1] 0.7298
> pnorm(0.8988)
[1] 0.8157
```

   c) **(6 points)** What is the probability that the sample mean lifetime falls between **520** and **550** minutes?

$$P(520 < \overline{X}_{\mathrm{L}} < 550) \approx P\left(\frac{520 - 540}{\frac{81}{\sqrt{53}}} < Z < \frac{550 - 540}{\frac{81}{\sqrt{53}}}\right)$$

$$= P(-1.7976 < Z < 0.8988)$$

$$= \Phi(0.8988) - \Phi(-1.7976)$$

$$= 0.8157 - 0.0361 = 0.7796$$

d) **(12 points)** Suppose the testing software is programmed to automatically discard any batch of **53** AirBuds if their sample mean is lower than **515** minutes, thus a batch is successfully retained and recorded if their sample mean is **at least 515** minutes. Given that a batch **is successfully recorded**, what is the probability that its sample mean falls **between 520** and **550** minutes?

$$P(520 < \bar{X}_L < 550 | \bar{X}_L > 515) = \frac{P\left(\frac{520-540}{\frac{81}{\sqrt{53}}} < Z < \frac{550-540}{\frac{81}{\sqrt{53}}}\right)}{P\left(Z > \frac{515-540}{\frac{81}{\sqrt{53}}}\right)}$$

$$= \frac{P(-1.7976 < Z < 0.8988)}{P(Z > -2.2469)}$$

$$= \frac{\Phi(0.8988) - \Phi(-1.7976)}{1 - \Phi(-2.2469)} = \frac{0.7796}{1 - 0.0123} = 0.7893$$

4. **(26 points)** In an effort to consolidate the conceptual understanding of interval estimation (*confidence interval and bounds*) in STAT350, an interactive trivia-based classroom activity was implemented during the Fall semester. Students were asked to estimate the sale price (in US dollars) of the first item ever sold on eBay, which was a broken laser pointer. A random sample of $n = 16$ student point estimates yielded a sample mean of $\bar{x} = 13.22$ and a sample standard deviation of $s = 4.3$. Typically, this type of guessing data tends to be mildly skewed due to psychological anchoring in human decision-making.

   a) **(2 points)** Does the information provided above justify the construction of standard confidence intervals or bounds? Give one statistical justification for your answer.

Two possible choices:

Yes, justified: Yes. The sample is described as a random sample, which satisfies the independence assumption. Although the data are mildly skewed and $n = 16$ is small to moderate size, the t-procedure is robust to mild departures from normality. With only mild skewness, the sampling distribution of $\bar{X}$ should be approximately normal even at this moderate sample size, so constructing a t-based confidence interval or bound is reasonable.

No (independence concern): The estimates were collected during a classroom activity, so students may have influenced each other's guesses through verbal or nonverbal cues. Even though the problem states a random sample, the shared environment may violate the independence assumption required for t-procedures. Additionally, the size of the classroom might not be sufficiently large to treat the samples as independent.

No (normality concern): Independence is satisfied by the random sampling, but the population is stated to be slightly skewed and $n = 16$ is not large enough for the CLT to guarantee an approximately normal sampling distribution.

b) Suppose we assume the conditions are met to conduct statistical inference. The actual selling price of the broken laser pointer was $14.83. Suspecting that students tend to underestimate, the instructor would like to establish, with 90% confidence, a confidence upper bound for the true mean of all student guesses across STAT 350 during the fall semester.

i. **(3 points)** Which R output below provides the correct critical value for this computation?

Ⓐ
```
> qnorm(0.05,lower.tail = FALSE)
[1] 1.644854
```

Ⓑ
```
> qt(0.1, df = 15,lower.tail = FALSE)
[1] 1.340606
```

Ⓒ
```
> qnorm(0.1,lower.tail = FALSE)
[1] 1.281552
```

Ⓓ
```
> qt(0.05, df = 15,lower.tail = FALSE)
[1] 1.75305
```

ii. **(5 points)** Compute and <u>**interpret**</u> the 90% confidence upper bound in the context of the problem.

$$\left(-\infty, \bar{x} + t_{0.1,df=15} \cdot \frac{4.3}{\sqrt{16}}\right)$$

$$(-\infty, 13.22 + 1.340606 \cdot 1.075)$$

$$(-\infty, 14.6615)$$

The **90% confidence upper bound** is $\mu < 14.6615$.

We are 90% confident that the true mean μ of all student guesses regarding the value at which the broken laser pointer was sold across STAT 350 during the fall semester is bounded below $\mathbf{14.6615}$.

c) **(2 points)** In general, t-intervals/bounds are [_______________] their corresponding z-intervals/bounds (assuming the same confidence level, standard error, and sample size). Select the correct option to fill in the blank.

Ⓐ narrower than

Ⓑ wider than

Ⓒ the same width as

d) Recently, a digital marketing analyst claimed that the true mean price for vintage RadioShack laser pointers is \$225. The prices of these vintage pointers are known to be normally distributed with a population variance of $\boldsymbol{\sigma^2 = 64}$. A Purdue engineering historian believes the true mean price is different. He collects a random sample of $\boldsymbol{n = 12}$ prices from e-commerce platforms, yielding a sample mean of $\boldsymbol{\bar{x} = 210}$ and a sample standard deviation of $\boldsymbol{s = 15}$. Use a significance level $\boldsymbol{\alpha = 0.02}$ for all inference calculations below.

i. **(4 points)** Provide the first two steps of the four-step hypothesis testing procedure.

Step 1: Define the parameter(s) of interest

$\boldsymbol{\mu}$ is the true mean price for vintage RadioShack laser pointers in US dollars

Step 2: State the hypothesis

$$\boldsymbol{H_0: \mu = 225}$$

$$\boldsymbol{H_a: \mu \neq 225}$$

ii. **(3 points)** Compute the appropriate test statistic.

$$\boldsymbol{Z_{TS} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{210 - 225}{8/\sqrt{12}} = -6.4952}$$

iii. **(3 points)** Which of the following R outputs below is appropriate for computing the $\boldsymbol{p}$**-value** for this test?

Ⓐ
```
> 2*pt(abs(t_ts),df=11,lower.tail=FALSE)
[1] 0.005294732
```
Ⓑ
```
> 2*pnorm(abs(z_ts), lower.tail=FALSE)
[1] 8.292839e-11
```
Ⓒ
```
> pnorm(abs(z_ts),lower.tail=FALSE)
[1] 4.14642e-11
```
Ⓓ
```
> pt(abs(t_ts),df=11,lower.tail=FALSE)
[1] 0.002647366
```

iv. **(4 points)** State your decision and provide a conclusion in the context of the problem.

Since, the $\boldsymbol{p}$-value $= \boldsymbol{8.292839e - 11}$ is less than $\boldsymbol{\alpha = 0.02}$, we have evidence to reject the null hypothesis.

The data does give **strong** support ($\boldsymbol{p}$**-value** = $\boldsymbol{8.292839e - 11}$) to the claim that the true mean price for vintage RadioShack laser pointers in US dollars differs from the \$225 claimed by the analyst.

5. **(26 points)** Researchers at a Purdue agricultural extension are evaluating two **soil amendments**, the standard treatment ($\mathbf{S}$) and a new organic treatment ($\mathbf{N}$), to determine whether the new treatment increases soybean yield (measured in bushels per acre). They selected 18 farm plots in Tippecanoe County. Each plot is split in half: one half receives Amendment $\mathbf{S}$ and the other half receives Amendment $\mathbf{N}$. After the growing season, soybean yield is recorded for each half-plot.
The researchers calculate the difference for each plot: $\mathbf{D} = \mathbf{N} - \mathbf{S}$. They have verified that the distribution of these differences is approximately normal.

	Amendment N	**Amendment S**	D = N − S
n	18	18	18
Sample Mean	54.8	51.2	3.6
Sample Std Dev	7.3	6.9	5.4

a) **(2 points)** Which testing procedure is appropriate for this experiment?
Ⓐ Two-sample independent $\boldsymbol{t}$-test
Ⓑ Two-sample paired $\boldsymbol{t}$-test

b) **(6 points)** Explain what specific characteristic(s) in the experimental design motivated your choice of testing procedure in part **a)**.

Each plot is split in half and as such the two measurements share the same plot (soil, moisture, sunlight, etc…), creating a natural pairing.

c) **(4 points)** Provide the first two steps of the four-step hypothesis testing procedure. Use a significance level of $\boldsymbol{\alpha} = \mathbf{0.03}$.

Step 1: Define the parameter(s) of interest
$\boldsymbol{\mu_D}$ is the true mean difference of soybean yield between land treated with the new organic treatment ($\mathbf{N}$) and that of standard treatment ($\mathbf{S}$). Here the difference $\mathbf{D}$ is defined as $\mathbf{D} = \mathbf{N} - \mathbf{S}$.
Step 2: State the hypothesis

$$H_0: \mu_D \leq 0$$

$$H_a: \mu_D > 0$$

Also accept

$$H_0: \mu_D = 0$$

$$H_a: \mu_D \neq 0$$

d) **(6 points)** Calculate the test statistic for this experiment. Show your work.

$$t_{TS} = \frac{\bar{d} - 0}{s_D/\sqrt{n_D}} = \frac{3.6}{5.4/\sqrt{18}} = 2.8284$$

e) **(3 points)** Select the most appropriate **R** command to compute the p-value for this specific test.

B or C has to be consistent with hypothesis in part c).

Ⓐ `pt(test_statistic, df = 17, lower.tail = TRUE)`

Ⓑ `2*pt(abs(test_statistic), df = 17, lower.tail = FALSE)`

Ⓒ `pt(test_statistic, df = 17, lower.tail = FALSE)`

Ⓓ `pt(test_statistic, df = 34, lower.tail = FALSE)`

Ⓔ `pt(test_statistic, df = 30.27, lower.tail = FALSE)`

Ⓕ `pnorm(test_statistic, lower.tail = FALSE)`

f) **(5 points)** The p**-value** for the correct test was found to be **0.0058**. Using a significance level of $\alpha = 0.03$, state your formal decision and write a conclusion in the context of the problem.

Since, the p-value $= 0.0058$ is less than $\alpha = 0.03$, we have evidence to reject the null hypothesis.

The data **does** give **strong** support (p**-value** = **0.0058**) to the claim that the true mean difference of soybean yield between land treated with the new organic treatment ($\mathbf{N}$) and that of standard treatment ($\mathbf{S}$) is positive, thus indicating the new organic treatment has a positive effect on crop yield.

Note: The last statement is dependent on the choice of hypothesis.